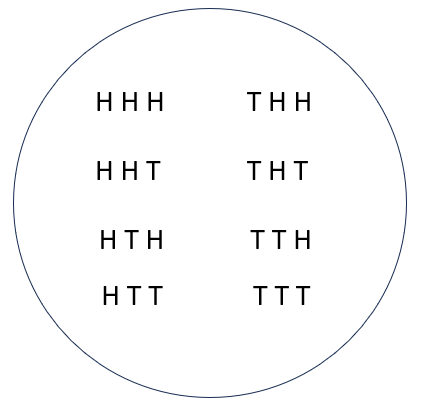
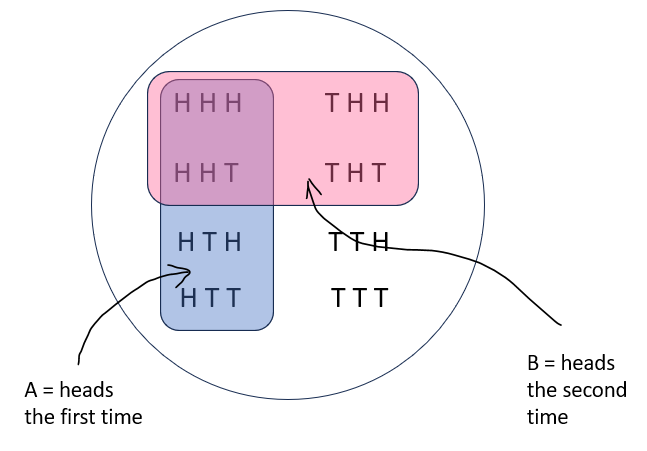
**Event Composition**

It’s advantageous to decompose a universal set into a sample space – a set of all possible mutually exclusive outcomes. Each of these outcomes can be assigned a probability. An event would be considered a subset of these outcomes corresponding to some, well, ‘event’. The probability of the event would just be the sum of the probabilities of all the outcomes that comprise the event. For example, consider three coin tosses. The sample space would be:



We can identify events. For instance, getting heads on the first toss and getting heads on the second toss would be the blue and red guys, respectively:



**Probability**

So we have, just as always,



In example, can see these probabilities are P(A) = P(B) = ½.

**Joint Probability**

So say we have a composite probability distribution P(a,b), or P(a,b,c), etc. This is the often written as just P(ab) or P(abc) and is the joint probability distribution of the events. It is the probability that a and b occur, or that a and b and c occur. Knowledge of this allows us to construct all other probabilities of interest.



So in the example above, we can see P(AB) = 2/8 = ¼.

**Disjunctive Probability**

So P(ab) gives the probability of a and b jointly occuring. What about the probability of either or occuring? This is:



And if we think about it,



Kind of follows from the Boolean set stuff. So altogether:



etc. And we’ll note that if events are non-overlapping, then these simplify to:



In example above, we can see P(A⋃B) = P(A) + P(B) – P(AB) = ½ + ½ - ¼ = ¾, which is true.

**Conditional Probability**

We can work out the probability of b given, well, a, say. From coin toss diagram above, we could say:



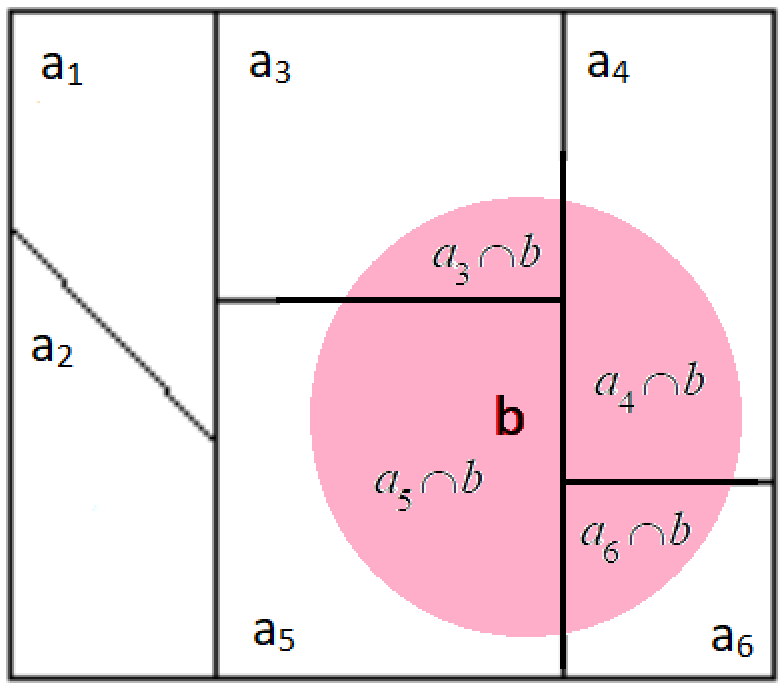
So,



In our example up above, we can see that P(B|A) = ½, and we get this from our formula too = P(A|B) = P(AB)/P(A) = (1/4)/(1/2) = ½. Another way to write this, from Stat Quest, is:



This makes the division by P(a) a more intuitive. So P(b|a) would just be P(a,b) with *a* held constant to its given value so to speak. Except, this wouldn’t be normalized. So we’d say P(b|a) = CP(a,b). And C can be worked out by demanding ΣbP(b|a) = 1 → CΣbP(a,b) = 1 → CP(a) = 1 → C = 1/P(a). So then we have P(b|a) = P(ab)/P(a). If b doesn’t depend on a at all, then we’d expect P(b|a) to be the same as P(b). And this would imply that P(ab) = P(a)P(b), i.e., that a and b are independent variables. Might be easier to see geometrically. So consider a universal set of events aj=1,2,3,4,5,6. And some other subset event b. Note the event would take up the rest of the space, i.e., the white space.



Can see that P(b|a3) is just the area of a3 ∩ b, divided by the area of a3. We can use the conditional probability formula to write a nice formula for the joint probability.



And we can generalize this formula to more variables. For instance,



and even more generally.



For concision, we might write this as:



Finally, we’ll note that if events are independent, then, meaning P(a|previous) = P(a), then this simplifies to:



Going back to our example, we see that A and B are independent events because P(A|B) = P(A). So we should have P(AB) = P(A)P(B) = (1/2)(1/2) = ¼, which is true.

**Marginal Probability**

By summing over all possible values of b, say, we can get the probability distribution of just a by itself.



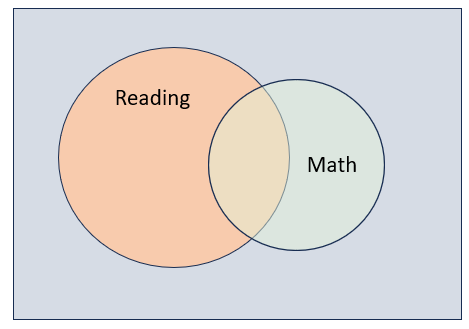
So we can say:



where in the latter we presume b can take just two values? Well not really. It would still hold regardless of the number of values b can take on. Going back to our example, we should have: P(A) = P(AB) + P(A) = ¼ + ¼ = ½.

**Example**

A survey of 120 elementary school children found that 30% of them liked math only, 35% liked reading only, and 25% liked both. Everybody liked recess. What percentage of students liked math? What percentage of them didn’t like reading? What percentage of them liked only recess? What percentage liked recess, but not reading? Making a Venn diagram thing out of it,



We can see that those who like math constituted the math exclusive part + the intersection. So,



Could also do the Boolean manipulation,



Likewise,



The probability of someone liking either math or reading is:



So 0.10 of them only like recess.



Percentage that likes recess but not reading is just 1 – P(Reading). P(Reading) = P(Reading only) + P(Reading and Math). so we have 1 – P(Reading only) – P(Reading and Math) = 1 – 0.35 – 0.25 = 0.40.

**Example**

What are the odds that in a room full of 30 people, no two people share the same birthday? We could look for probability that each time you pick a person, they don’t have the same birthday as any of the previous people. The first pick is free. The second person can have 364 possible birthdays. The third person can have 363 possible birthdays, etc.

Well we could say it’s the probability:



**Example**

Say we flip a coin r times and record the sequence. How many times, n, would we have to repeat r coin flips to have a p = 40% chance of getting that sequence again? Let’s look, rather to the odds of not getting that sequence ever, in n sequences. This will be:



This is the same as we got in the combinatorics file.

**Example**

Resistors A, B, and C have a 5% chance of shorting. Resistor A and B are connected in parallel, and together in series with C. These are then connected to a switch and a battery. What is the probability that when the switch is closed, current will flow?

So we need both C and either of A and B to not short for current to flow. So what is:



Well these are independent events so we can say,



**Example**

Out of a class of 30, 3 people are chosen at random to forego recess and weed the garden, sweep the sidewalks, clean the classroom. Say there are 13 boys and 17 girls in the class. What is the probability that all three students chosen are girls? Well we’d have:

So the probability is:



**Example**

Twenty people throw their names in a hat, and three people are chosen. What is the probability a given person is chosen? So,



which is is just as we found in the Combinatorics section.

**Example**

An employer has a pool of 30 applicants. What are the odds that if she chooses 10 to interview she will get at least one of the top 3 candidates? This should be:



which is what we got last time too. But I guess we’ll note that:



**Example**

A bag contains 5 different fruits. You choose 3 out of the 5 fruits, and you do this 3 different times. What are the odds you choose grapes each time?

